Extension of Second Level Adaptation using Multiple Models to SISO Systems

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***Abstract*— Second level adaptation using multiple models for the adaptive control of linear systems with unknown parameters was introduced in [1], and its robustness properties were discussed in [2]. In both cases , the plant, the identification models, and the reference model were assumed to be described by state equations in companion form with all state variables accessible.**

To make the approach more widely applicable (and hence practically more attractive), an attempt is made in this paper to extend the same concepts to more general systems. The latter include systems whose state variables are accessible, but whose A matrices are in general form, and SISO systems.

As in earlier papers, simulation results are included wherever appropriate to demonstrate that significant improvement in performance can be achieved when parametric uncertainty in the system is large or when noise and time variations are present.

I. Introduction

Over the past two decades, multiple-model based adaptive control has been studied for the control of systems with large parameteric uncertainties. During this period, two general methodologies referred to as "Switching"[3] and "Switching and Tuning"[4] emerged for the identification and control of unknown linear systems. In the former, the best among a set of fixed models is chosen at every instant, based on a performance criterion, and is used to control the unknown plant. In the latter case, an adaptive system is initiated with the controller parameters at the same values in parameter space as in "switching", but are adjusted adaptively. These, in turn, are used to control the system. In both cases, it is seen that control is based on a single model.

In [1], a new approach was proposed termed second level adaptation, in which the responses of multiple models were used to determine the control parameter at every instant. By avoiding switching between models, shorter transition times and smoother responses were achieved. The stability of the overall system was also demonstrated. In [2], the performance and robustness of the scheme proposed in [1] was discussed and due to the space limitations, possible extensions to more general systems were merely outlined. Simulation studies presented in both [1] and [2] demonstrated clearly that much faster and more robust adaptation could be achieved using second level adaptation.

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In spite of the advantages mentioned above, one of the principal shortcomings of [1] and [2] is that in both papers the unknown plant is assumed to be described by a differential equation in companion form. This limits significantly the applicability of the approach, since most practical system do not satisfy the above requirement. Hence, in this paper, an attempt is made to extend the result in [1] and [2] to more general systems. These include systems with all state variables acessible but whose A matrices have general forms, and single-input single-output (SISO) systems where control has to be effected using only input and output data from the plant. These extensions permit the results to be applied to a substantially larger class of plants making them practically attractive. As in [1] and [2], simulation studies are included throughout the paper, and indicate that the approach is significantly better than conventional adaptive control, particularly when the uncertainty in the plant is large.

II. Mathematical Preliminaries and Previous Results

In section IIA, we first review briefly the reasons for using multiple models for improving the performance of a simple adaptive system which attempts to estimate a set of parameters in an algebraic system. Following this, in Section IIB, the principal results contained in papers [1] and [2] are summerized for use in the following sections. Due to space limitations, the explanations given in this section are of necessity succinct.

A. Mathematical Preliminaries

Let $\theta_p \in \mathbb{R}^n$ be an unknown constant parameter vector to be estimated using the input output data provided by the equation:

$$\theta_p^T u(t) = y(t) \tag{1}$$

where $u(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}$ are respectively the available input and output.

1) First Level Adaptation: The standard procedure in adaptive control theory is to set up a model

$$\hat{\theta}_p^T(t)u(t) = \hat{y}(t) \tag{2}$$

and adapting $\hat{\theta}_p(t)$ using the error e(t)=$\hat{y}(t)$-y(t) and the input u(t) based on the adaptive law

$$\dot{\hat{\theta}}_p(t) = \dot{\tilde{\theta}}_p = -e(t)u(t) = -u(t)u^T(t)\tilde{\theta}(t) \tag{3}$$

where $\hat{\theta}_p(t) - \theta_p = \tilde{\theta}_p(t)$ and e(t)=$\tilde{\theta}_p^T(t)u(t)$. It immediately follows that $\tilde{\theta}_p(t)$ (and hence $\hat{\theta}_p(t)$) is bounded and

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$\tilde{\theta}_p(t) \to 0$ or $\hat{\theta}_p(t) \to \theta_p$ as $t \to \infty$ if u(t) is persistently exciting. A scalar γ, or constant (or time-varying) symmetric matrix Γ (or Γ(t)) can also be used in the adaptive law to improve the speed of response.

2) Second Level Adaptation: If $\theta_p \in \mathscr{S} \subset \mathbb{R}^2$ in the parameter space where $\mathscr{S}$ is a compact set, three parameter vectors are initiated at $\theta_1(0)$, $\theta_2(0)$, $\theta_3(0)$ such that θ_p lies in their convex hull. As in the previous case, $\theta_i(t)$ is adjusted using first level adaptive laws described earlier so that $\dot{\theta}_i(t) = e_i(t)u(t)$ where $e_i(t) = \theta_i^T(t)u(t) - y(t)$. Since $\sum_{i=1}^{3} \alpha_i\theta_i(t_0) = \theta_p$ with $0 \le \alpha_i \le 1$, $\sum_{i=1}^{3} \alpha_i = 1$, it follows that θ_p is also in the convex hull of $\theta_i(t)(i = 1,2,3)$ for all $t \ge 0$, when the intitial values of the adaptive parameters are $\theta_1(0) = [5,10]$,$\theta_2(0) = [-5,-5]$,$\theta_3(0) = [10,-5]$. This is shown in Figure (1).

Comment: Note that second level adaptation converges to $\theta_p = [4,4]$ in a very short time compared to the time of convergence of the first level adaptive parameters.

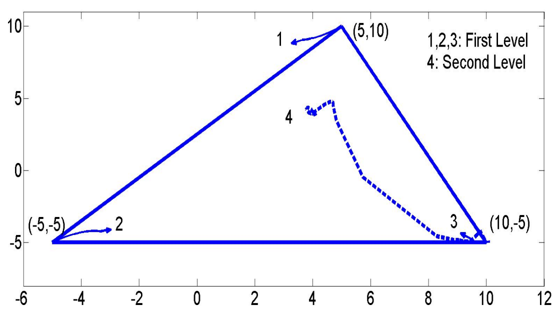

Fig. 1: First and Second Level Adaptation

Second level adaptation consists in using the estimates $\theta_i(t)$ obtained from first level adaptation to determine an estimate of θ_p. Since $\sum_{i=1}^{3} \alpha_i\theta_i(t) = \theta_p$, it follows that $\Sigma\alpha_i(t)e(t) = 0$. If the constants α_i can be estimated, they can be considered as an alternative parameterization of the unknown vector θ_p. Determinining $\alpha_i(i = 1,2,3)$ is termed second level adaptation.

Since $\sum_{i=1}^{3} \alpha_i(t)e(t) = 0$ and $\alpha_3 = 1 - \alpha_1 - \alpha_2$, we have $\alpha_1[e_1(t) - e_3(t)] + \alpha_2[e_2(t) - e_3(t)] = -e_3(t)$, or alternatively the algebriac equation:

$$E(t)\alpha = -e_3(t) \tag{4}$$

where $E(t) = [e_1(t) - e_3(t), e_2(t) - e_3(t)]$ and $\alpha^T = [\alpha_1, \alpha_2]$. To estimate the constant vector α, the differential equation

$$\dot{\hat{\alpha}}(t) = -E^T(t)E(t)\hat{\alpha} - E^T(t)e_3(t) \tag{5}$$

is solved. The convergence of the parameter vector $\hat{\alpha}(t)$ to the constant α is shown in Figure.2

Comments:

(i) The additional information that the unknown parameter vector θ_p belongs to a convex set in parameter space is needed for second level adaptation.

(ii) The adavantages of second level adaptation are evident even at this simple level. The unknown parameter vector α is defined by a linear algebraic equation and estimated using a linear differential equation. The convergence is consequently much faster than in first level adaptation.

(iii) If noise is present in the original equation, it does not affect the matrix E(t) but only the error $e_3(t)$ makeing analysis substantially simpler.(note that the elements of E(t) are $e_i(t) - e_3(t)$)

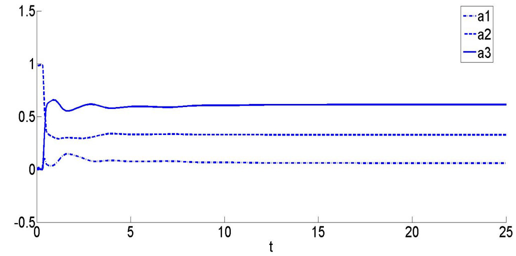

Fig. 2: different α values

3) Second Level Adaptation with Fixed Parameters: In the previous case, we assumed that all parameters $\theta_i(t)$ are adapted and hence time varying. For many situations it is found preferable to choose θ_i to be constant. This does not affect the arguments used thus far.

B. Previous Results

The results derived in [1] can be stated briefly in this section using the discussions in section IIA.

1) The Problem: A linear time-invariant plant Σ_p, with unknown parameters, is described by the differential equation

$$\Sigma_p : \dot{x}_p = A_p x_p + bu \quad x_p(t) \in \mathbb{R}^n \tag{6}$$

where (A_p,b) is in controllable canonical form. A stable reference model Σ_m and N stable identification models Σ_i are respectively described by the equations:

$$\Sigma_m : \dot{x}_m = A_m x_m + bu \tag{7}$$

$$\Sigma_i : \dot{x}_i = A_m x_i + b[\theta_i^T x_p + u](i = 1,2,...,N) \tag{8}$$

where A_m is stable and in companion form. The last rows of the matrices A_m and A_p are respectively a_m^T and a_p^T, where a_m is at the discretion of the designer and a_p is the unknown parameter vector to be estimated.

2) First Level Adaptation: In conventional adaptive control theory, using a single model (i.e. i=1) $\theta_1(t)$ is adjusted as $\dot{\theta}_1(t) = -e^T Pbx_p(t)$ where $A_m^T P + PA_m = -Q < 0$, and this assures that all signals are bounded. Further, if the input is persistently exciting, $\lim_{t\to+\infty} \theta_1(t) = a_p - a_m$from which a_p may be computed.

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3) Second Level Adaptation: As in section IIA, N=n+1 adaptive models are initialized with parameter values $\theta_i(0)$ such that a_p lies in the convex hull of $a_m + \theta_i(0)$, and adapted using the adaptive laws $\dot{\theta}_i(t) = -e_i^T P b x_i(t)$ where $e_i(t) = x_i(t) - x_m(t)$ is the state error vector of the i^{th} model. Since the plant state vector $x_p(t)$ is accessible, all the adaptive models are initialized with $x_i(0) = x_p(0)$, or alternatively $e_i(0) = 0$. Once again, using the linear dependence of the output error vectors $e_i(t)$ on the parameters $\theta_i(t)$, it is shown that:

a) θ_p lies in the convex hull of $a_m + \theta_i(t)$ for all $t \geq 0$; $i \in \Omega = \{1, 2, ..., n+1\}$

b) $\sum_{i=1}^{n+1} \alpha_i e_i(t) = 0$; $\sum_{i=1}^{n+1} \alpha_i = 1; 0 \leq \alpha_i \leq 1$

which in turn yields

$$E\alpha = -e_{n+1}(t) \tag{9}$$

where E(t) is the matrix $E(t) = [e_i(t) - e_{n+1}(t), ..., e_n(t) - e_{n+1}(t)]$. $\alpha = [\alpha_1, \alpha_2, ...\alpha_n]^T$ is the constant parameter vector to be estimated, and represents an alternative parameterization of the plant. $\hat{\alpha}(t)$, the estimate of α satisfies the vector differential equation

$$\dot{\hat{\alpha}}(t) = -E^T(t)E(t)\hat{\alpha}(t) - E^T(t)e_{n+1}(t) \tag{10}$$

As stated in section IIA, $\alpha(t)$ converges significantly faster than $\theta_i(t)$ and enjoys the advantage of linearity over first level adaptation, whose adaptive laws are seen to be clearly nonlinear (products of $e_i(t)$ and $x_p(t)$)

As in section IIA, fixed identification models can also be used and enjoy all the advantages of second level adaptation including rapid convergence and robustness in the presence of input disturbance or output noise.

In the following Sections, these concepts are extended to more general systems.

III. General Systems with State Variables Accessible

As summerized in Section II, second level adaptation described in [1] was applied primarily to linear plants whose A matrices are in companion form, and all of whose state variables are accessible. In this section we first discuss briefly the identification and control problems when the A matrix is not in companion form and when the b vectors are also unknown. Following this, we provide reasons for attempting only the identification problem in this section, using second level adaptation. In section II, as described in [1], [2], the pair [A,b] is in controllable canonical form and hence the reference model and the identification models were also chosen to have this property. This, in turn, made the identification and control problems relatively straight forward. However, if the plant Σ_p is described by the differential equation

$$\Sigma_p : \dot{x}_p = Ax_p + bu \tag{11}$$

where A and b are a general constant matrix in $\mathbb{R}^{n\times n}$ and a vector in $\mathbb{R}^n$ respectively, there is no simple procedure for setting up a stable reference model, whose state variables the given plant can track.

The best that one can hope for is tracking some of the state variables of the reference model by the corresponding variables of the plant. Since these are precisely the problems considered in the following sections, we limit ourselves to the identification problem stated below:

<u>The Identification Problem:</u> Given a stable plant Σ_p described by the equation (11) in which (A,b) are not in controllable canonical form, if some of the elements of matrix A and vector b are unknown, determine how second level adaptation can be used to identify them.

If a_{ij} and b_l are $N(\leq 2n)$ parameters which are unknown, the problem, as in [1] and [2] is to estimate them as $a_{ij}(t)$ and $b_l(t)$ in a stable fashion so that when the input is persistently exciting $\lim_{t\to+\infty} \hat{a}_{ij}(t) = a_{ij}$ and $\lim_{t\to+\infty} \hat{b}_l(t) = b_l (i, j, l = 1, 2, ..., n)$.

Let A_m be a matrix and b_m be a vector which have the same known elements as A and b, and in place of the unknown elements a_{ij} and b_l, constants a_{ij}^* and b_l^*. The constants a_{ij}^* are chosen such that A_m is stable (such values exist, since the plant was assumed to be stable). In such a case a typical identification model Σ_i can be chosen as

$$\Sigma_i : \dot{\hat{x}}(t) = A_m \hat{x}(t) + [A(t) - A_m]x_p(t) + [b_m + \psi(t)]u \tag{12}$$

Our objective is to adjust the unknown parameters of the matrix A(t) and the vector $\psi(t)$, using the output error $e(t) = \hat{x}(t) - x(t)$, so that the overall system is stable and e(t) tends to zero. It can be readily shown that Σ_i can also be represented as

$$\Sigma_i : \dot{\hat{x}}(t) = A_m \hat{x}(t) + b_m u(t) + \Sigma b_i \tilde{\theta}_i(t) z_i(t) \tag{13}$$

where b_i are known constant vectors, $z_i(t)$ are elements of the state vector $x_p(t)$ or the input u(t), and $\tilde{\theta}_i(t)$ are the parametric errors. Since the forms of the reference model Σ_m and the identification model Σ_i are known, the error equation and adaptive laws for first level adaptation can be expressed as

$$\dot{e}(t) = A_m e(t) + \Sigma b_i \tilde{\theta}_i(t) x_p(t) \tag{14}$$

$$\dot{\tilde{\theta}}_i = -e^T P b_i x_p(t) (i = 1, 2, ...N) \tag{15}$$

The extension of second level adaptation to the above problem is straight-forward from this point. $\theta_i(t_0) (i = 1, 2, ...n+1)$ are chosen so that $\Sigma b_i \theta_i^*$ is in its convex hull, and the vector α is estimated as described in section II. To clarify the concepts presented, simulation results using a simple example are included below.

Example 1 *A stable plant is described by the equation (16) where one parameter $a_{31} = -30.48$ in A and one parameter $b_3 = 8$ in b are assumed to be unknown, and need to be estimated.*

$$\dot{x}_p = Ax_p + bu \tag{16}$$

where $A = \begin{bmatrix} -11.62 & -15.73 & 9.47 \\ -15.38 & -24.27 & 14.52 \\ -30.48 & -47.92 & 27.89 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$

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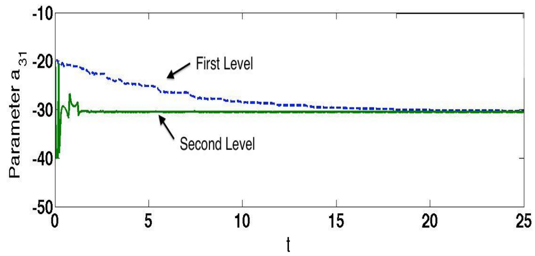
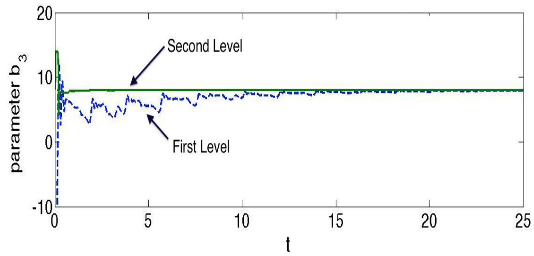


(a) Parameter a_{31}=-30.48



(b) Parameter b_3=8

Fig. 3: Identification using First Level and Second Level Adaptation

It is assumed that the unknown parameters satisfy the inequaties $-60 \leq a_{31} \leq -20$ and $4 \leq b_{31} \leq 10$. Three fixed models were chosen where all the parameters of A and b other than a_{31} and b_3 are the same as those of the plant. The values of a_{31} and b_3 were chosen so that the uncertainty region is contained in their convex hull. These values were chosen to be (-20,2),(-80,2) and (-20,26).

At this point we note that these values represent the initial values when adaptive models are used, but the location of the fixed models, when no first level of adaptation is used with multiple models. Both first and second level adaptive control was carried out and once again, estimation using second level adaptation is seen to be substantially better both with respect to speed and accuracy.

The simulation results using both first level and second level apaptation are shown in Figure (3a) and Figure (3b).

IV. Single Input and Single Output System

The problems considered thus far, in which the state variables of the unknown plant are accessible, are relatively simple problems of adaptive control. For a proper comparision of conventional and second level adaptation, it would be more appropriate to consider the standard adaptive control problem of an SISO system in which the plant state variables are not accessible, but have to be estimated. This is considered in this section. It is found that many of the simple concepts developed for parameter estimation in Section 2, are directly applicable to this case.

A. Statement of the Problem

1) Plant: A single-input single-output plant Σ_p is described by the state equations

$$\Sigma_p : \dot{x} = Ax + bu; \quad y = cx \tag{17}$$

where $A \in \mathbb{R}^{n\times n}$, $b, c^T \in \mathbb{R}^n$, and the input u(t) and output y(t) are scalar functions of time. The parameters of A,b and c are assumed to be unknown and the transfer function $W_p(s) = C[SI-A]^{-1}b$ is given by

$$W_p(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n} \tag{18}$$

where the 2n coefficients b_i and a_i of the numerator and denominator polynomials are assumed to be unknown. It is further assumed that $W_p(s)$ has all its zeros in the open left-half of the complex plane.

As in standard adaptive control problems, it is assumed that the relative degree of the plant n^* (number of poles - number of zeros=n-m), and the sign of the high frequency gain (i.e. sgn b_1) are known, and that the zeros of the plant lie in the open left half plane (minimum phase condition). For case of exposition we consider a plant of relative degree unity (i.e. m=n-1) as given in equation (18) and $b_1 = 1$ in the following discussions.

When the plant is known to be stable, its parameters (include b_l) can be estimated using the procedures described in this section. However, when such prior information is not available, identification and control have to proceed concurrently, to assure the boundness of all the signals in the system. We first state below the identification and control problems seperately.

2) The Identification Problem: Given the input u(t) and the output y(t) of a stable plant Σ_p described by the state equations (17) and/or the transfer function (18), estimate the unknown parameters (a_i and b_j in equation (18) using one or more models so thtat ($\lim_{t\to+\infty} |y_i(t) - y(t)| = 0 \ \forall i$), where $y_i(t)$ is the output of the i^{th} model.

3) The Indirect Control Problem: Given a stable reference model Σ_m with a bounded reference input r(t) and an output $y_m(t)$, a minimum transfer function $W_m(s)$ with relatice degree n^* equal to that of the plant, determine identifiers Σ_i and the structure of a controller as well as the adaptive laws for adjusting its parameters using the estimates generated by Σ_i, such that all the signals of the overall system are bounded.

$$\lim_{t\to+\infty} |y_m(t) - y(t)| = 0 \tag{19}$$

The solution to the above problem was given in 1980, and will be referred to in this paper as first level adaptation [6]. If N($\geq$ n+1) identification models are used, and the information provided by them determines the control input to the plant, we shall refer to it as second level adaptation.

Both direct and indirect control for the aymptotic tracking of the outputs of a reference model are well known in the literature. Since the multiple-model based approach is an indirected one, we shall use the latter for control and compare the procedure with the approach based on second level adaptation.

B. Parameterization of the identification Model:

It is well known that the proper choice of the parameterization of the identification models is essential for the generation

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of stable adaptive laws. Structures for the identification of the unknown parameters of adaptive linear systems have been studied extensively in the adaptive control literature [5]. Two general non-minimal representations of arbitary n^{th} order linear systems are shown in Figure (4). P(s), Q(s) and R(s) are polynomials (where R(s) is assumed to be Hurwitz), and the overall transfer function of the two representations are respectively:

$$W(s) = \frac{P(s)}{R(s)+Q(s)} \qquad \text{(Representation I)}$$

$$W(s) = \frac{P(s)}{(s+\lambda)R(s)+Q(s)} \qquad \text{(Representation II)}$$

1) Representation I: In this representation, P(s) and Q(s) are of $(n-1)^{th}$ degree, while R(s) is of degree 'n'. Since P(s) and (R(s)+Q(s)) correspond to the numerator and denominator polynomials of W(s), the coefficients of P(s) correspond to the coefficients b_i(i=1,2,...n), while those of Q(s) are linearly related to a_i(i=1,2,...,n)

Representation I is partcularly suited when our objective is primarily the identification of the plant parameters. In such a case the structure of the identifier has the form shown in Figure (5).

Two identical filters with inputs u and y are described by the equations:

$$\dot{\bar{\omega}}_1 = \bar{\Lambda}\bar{\omega}_1 + \bar{l}u \quad \dot{\bar{\omega}}_2 = \bar{\Lambda}\bar{\omega}_2 + \bar{l}y \tag{20}$$

generate the signals $\bar{\omega}_1(t)$ and $\bar{\omega}_2(t) \in \mathbb{R}^n$, and $\hat{y}(t)$, the eatimate of the output y(t) of the plant is given by

$$\bar{\theta}_1^T\bar{\omega}_1 + \bar{\theta}_2^T\bar{\omega}_2 = \bar{\theta}^T\omega(t) = \hat{y}(t) \tag{21}$$

or $\hat{y}(t)$ is a linear combination of $\bar{\omega}_1$ and $\bar{\omega}_2$. $\bar{\Lambda}$ in equation (20) is a stable matrix with characteristic equation R(s) which is Hurwitz (for convenience it is chosen to have a companion form), and $\bar{l} = [0,0,...,1]^T$). As stated easlier, the elements of the parameter vectors $\bar{\theta}_1$ and $\bar{\theta}_2$ are linearly related to the parameters of the plant. For adaptive identification, $\bar{\theta}(t)$ is adjusted using the adaptive law:

$$\dot{\bar{\theta}}(t) = -e(t)\omega(t) \quad e(t) = \hat{y}(t) - y(t) \tag{22}$$

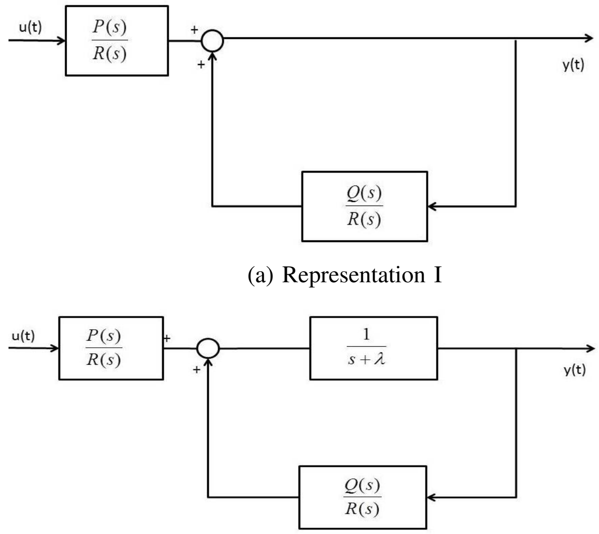
or

$$\dot{\bar{\theta}}_1(t) = -e(t)\omega_1(t) \quad \dot{\bar{\theta}}_2(t) = -e(t)\omega_2(t) \tag{23}$$

The simulation of the identification of a second order plant with 4 unknown parameters is considered in Example 2.

2) Representation II (Control): In this representation of the transfer function W(s), the forward path consists of a transfer function $\frac{1}{s+\lambda}$, where $\lambda > 0$ is known. Hence P(s), Q(s) and R(s) are $(n-1)^{th}$ degree polynomials with R(s) Hurwitz. Therefore $\frac{P(s)}{R(s)}$ and $\frac{Q(s)}{R(s)}$ are proper but not strictly proper transfer functions. This accounts for the structure of the identifier shown in Figure (6).

It is well known that for certainty equivalence adaptive control, the input to the plant is computed on-line using



(a) Representation I

(b) Representation II

Fig. 4: Two non-minimal representation

the estimates of the parameters of the plant. While both representations can be used towards this end, it is found to be substantially simpler if the reference model is embeded as part of the identifier as shown in what follows. In control problems, where the stability of the plant cannot be assumed, identification and control have to be carried out simultaneously, and for such an application, when the plant has a relative degree $n^* = 1$, representation II is prefered. The block diagram representation of this is shown in Figure (6). The filters in this case are described by the controllable pair $[\Lambda, l]$, where $\Lambda \in \mathbb{R}^{(n-1)\times(n-1)}$ and $l \in \mathbb{R}^{n-1}$. $\omega_1(t)$ and $\omega_2(t) \in \mathbb{R}^{n-1}$ are the outputs of the two filters (with u(t) and y(t) as the inputs respectively). Defining $\omega(t)$ and $\theta(t) \in \mathbb{R}^{2n}$ as

$$\omega^T(t) = [u(t), \omega_1^T(t), \omega_2^T(t), y(t)], \tag{24}$$

$$\theta^T(t) = [1, \theta_1^T(t), \theta_2^T(t), \theta_0(t)] \tag{25}$$

the estimate $\hat{y}(t)$ of the plant output y(t) can be expressed as

$$\hat{y}(t) = \frac{1}{s+\lambda}[\theta^T(t)\omega(t)] \tag{26}$$

and the adaptive laws for adjusting $\theta(t)$ are given by

$$\dot{\theta}(t) = -[\hat{y}(t) - y(t)]\omega(t) \tag{27}$$

The indirect adaptive control of an unstable second order system using the above approach is considered in Example 3.

C. First and Second Level Adaptation

Thus far we have considered the structures and the corresponding signals used for the identification of stable plants or the identification and control of unstable minimum

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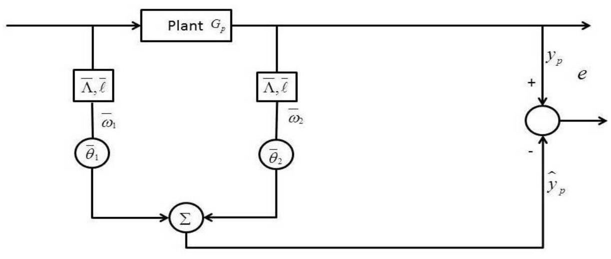


Fig. 5: Representation 1

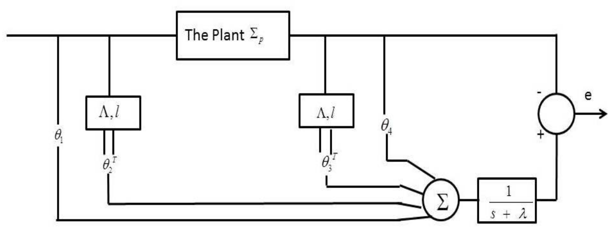


Fig. 6: Representation II

phase plants.

1) Identification: For identification in the first case, the signals $\bar{\omega}_1(t)$ and $\bar{\omega}_2(t) \in \mathbb{R}^n$ and the corresponding parameters $\bar{\theta}_1(t)$ and $\bar{\theta}_2(t) \in \mathbb{R}^n$ are used. For second level adaptation, the same signals $\bar{\omega}_1(t)$ and $\bar{\omega}_2(t)$ are used to generate the parameter estmates of (r+1) models (where r is the number of unknown parameters), which in turn are combined linearly to obtain the parameter estimates. The details of the procedure are provided in Example 2 for a second order system with 4 unknown parameters.

2) Control: For the control of an unstable system with relative degree unity, both first level and second level methods can be used. For first level, the parameters $\bar{\theta}_1(t), \bar{\theta}_2(t), \theta_0(t)$ are estimated using equations (22) and (23), and the input to the plant u(t) is obtained by feeding back the signal

$$-[\bar{\theta}_1^T(t)\omega_1(t) + \bar{\theta}_2^T(t)\omega_2(t) + \bar{\theta}_0(t)y(t)] = v(t) \tag{28}$$

so that u(t)=r(t)-v(t)

For second level adaptation, the same signals are used to generate the errors $e_1(t), e_2(t), ..., e_{n+1}(t)$ using multiple (fixed or adaptive models) and once again, the feedback signal is computed from the parameter estimates obtained. An example of the control of an unstable second order system having three unknown parameters using four fixed models is described in Example 3.

Example 2 *(Identification) A plant Σ_p has a transfer function*

$$W_p(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} \tag{29}$$

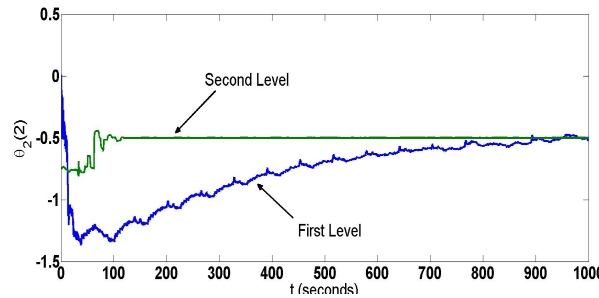
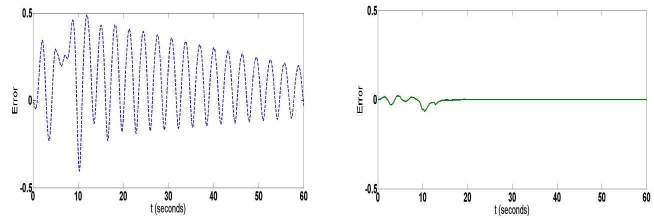


Fig. 7: Convergence of Parameter $\bar{\theta}_2(2)$



(a) First Level (b) Second Level

Fig. 8: Output Error for First and Second Level Adaptation (Identification)

where all four parameters ($b_1 = \frac{3}{2}$, $b_2 = \frac{1}{2}$,$a_1 = \frac{5}{2}$, $a_2 = \frac{3}{2}$) are assumed to be unknown. To estimate the parameters, Representation I was used with appropriate filter transfer functions.

1) First Level: The filter transfer function ($\bar{\Lambda}$, and $\bar{l}$) shown in Figure (5) has a transfer function $\frac{1}{(s+1)(s+2)}$. The parameter $\bar{\theta}_1(t)$ and $\bar{\theta}_2(t)$ after adaptation converged to the values $\bar{\theta}_1^T(\infty)$=[1.5,-2.5] and $\bar{\theta}_2^T(\infty)$=[0.5, -0.5], which can be used to compute the estimates of b_1, b_2, a_1, a_2 as $b_1 = \frac{3}{2}$, $b_2 = \frac{1}{2}$, $a_1 = \frac{5}{2}$, $a_2 = \frac{3}{2}$.

2) Second Level: Since four parameters are unknown, five fixed models were used for second level adaptation. The models are constant gain vectors $P_1 = [0, -2.5, 0.5, 1]$; $P_2 = [2, -2.5, 0, 0.5]$; $P_3 = [1.5, 0, 0.5, 0.5]$; $P_4 = [1.5, -5, -0.5, 0.5]$; $P_5 = [1.5, -2.5, -1, 0]$, which were chosen so that the unknown parameter vector lies in their convex hull. In the simulation, the convergence of a typical parameter $\bar{\theta}_2(2)$ is shown in Fugure (7) to compare first level and second level adaptations. While first level adaptation takes 900 seconds for the estimate to converge to the true value, it takes only 100 seconds with second level. The convergence of the output errors in the two cases are shown in Figure (8), and once again, second level is seen to be an order of magnitude better.

Example 2 deals with the identifaication of the parameters of a stable system. In the next example, using representation II, we attempt to control an unstable system with three unknown parameters.

Example 3 *(Control) An unstable plant Σ_p has a transfer*

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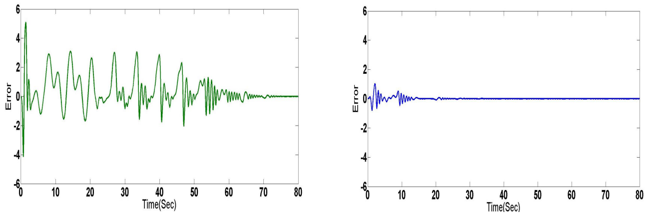


(a) First Level (b) Second Level

Fig. 9: Output Error for First and Second Level Adaptation (Control)

function

$$W_p(s) = \frac{s+1}{s^2-3s+2} \tag{30}$$

The high frequency gain of the plant is known to be unity, but the other three parameters $a_1 = -3, a_2 = 2, b_2 = 1$ are assumed to be unknown. The objective is to use an adaptive procedure for controlling Σ_p so that its output y(t) tracks the output of a reference model Σ_m with a transfer function $\frac{1}{s+1}$ and reference input $r = 10sin(5t) + sin(t)$. The convergence of the output error to zero using both first and second level adaptation is shown in Figure (9) and indicate that the latter is significantly faster.

V. ROBUSTNESS

In the previous section it was shown that the identification or control of an SISO system with second level adaptation is significantly faster than that of first level adpatation. The robustness of second level adaptation in the presence of noise and time-varying parameters has already been discussed in an earlier paper [2]. In this section, we merely present examples demonstrating these characteristics in an SISO system.

Example 4 *(Time-varying Parameters)*

Statement of the problem: A plant Σ_p which is unstable and has a time-varying parameter is described by the differential equation

$$\ddot{y} - (5 + 0.5sin0.2t)\dot{y} + 6y = \dot{u} + u \tag{31}$$

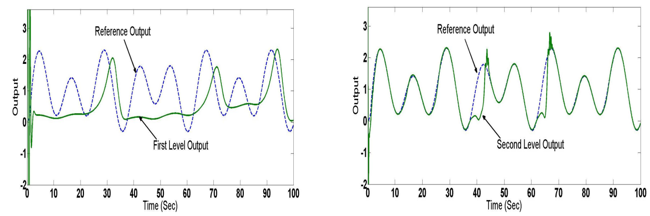
A reference model Σ_m has the transfer function $W(s) = \frac{1}{s+1}$ and a reference input r(t)=0.5sin(0.3t)+sin(0.5t)+1. The time-varying coefficient of $\dot{y}$ in the equation describing the plant (i.e.-(5+0.5sin0.2t)) is assumed to be unknown. The objective is to adaptively identify and control the plant so that the overall system is stable and the output y(t) tracks the output of the reference model asymptotically

Both first level and second level identification schemes were used to identify and control the unknown plant. The results are shown in Figures (10) and (11). Figure (10) shows the desired reference output and the response of the plant using first and second level adaptation. In Figure (11), the output errors are indicated.

Example 5 *(Unstable Plant with Noise) The example deals with the control of an unstable plant in the presence of output noise. The plant transfer function is*

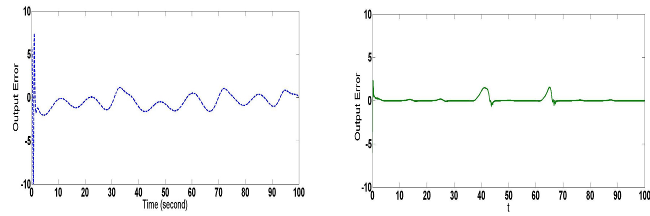
$$W_p(s) = \frac{s+1}{s^2-5s+6} \tag{32}$$

and as in the previous case, the three parameters ($b_2 = 1, a_1 = -5, a_2 = 6$) are assumed to be unknown. Additive Gaussian noise N(0,0.3) with zero mean and variance 0.3 is assumed to be present at the output and adaptation is carried out using both the methods discussed. The output errors using first and second level adaptationare shown in Figure (12) and clearly reveal the superiority of the latter.



(a) First Level (b) Second Level

Fig. 10: Adaptive Control of Plant with Time-varying Parameters - Plant and Model Outputs



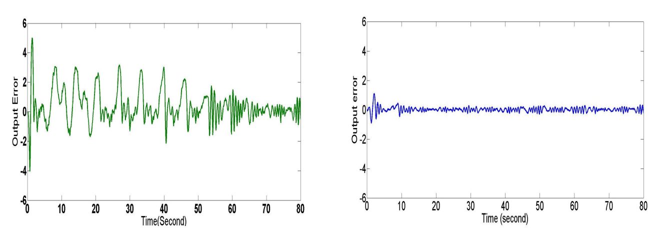
(a) First Level (b) Second Level

Fig. 11: Output Errors for Time-varying Parameters

VI. CONCLUSION

Second level adaptation was introduced in [1] for plants, identification models, and reference model that are described by state equations in companion form, when the state vector of the plant is accessible. In this paper, the concepts are extended to a much wider classes of systems, which make them considerably more attractive in practical applications.

Much of the paper is devoted to single-input single-output (SISO) systems where all adaptive decisions have to be made using only the information contained in the input and the output of the plant. After discussing the structure used



(a) First Level (b) Second Level

Fig. 12: Output Errors with Noise $N(0,0.3)$

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for identification, second level adaptation is applied to the control problem. It is shown that for the control of unstable systems with and without observation noise, and when the parameters vary with time, second level adaptation results in far better performance than conventional adaptive control.

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